FQL++ IDE Manual

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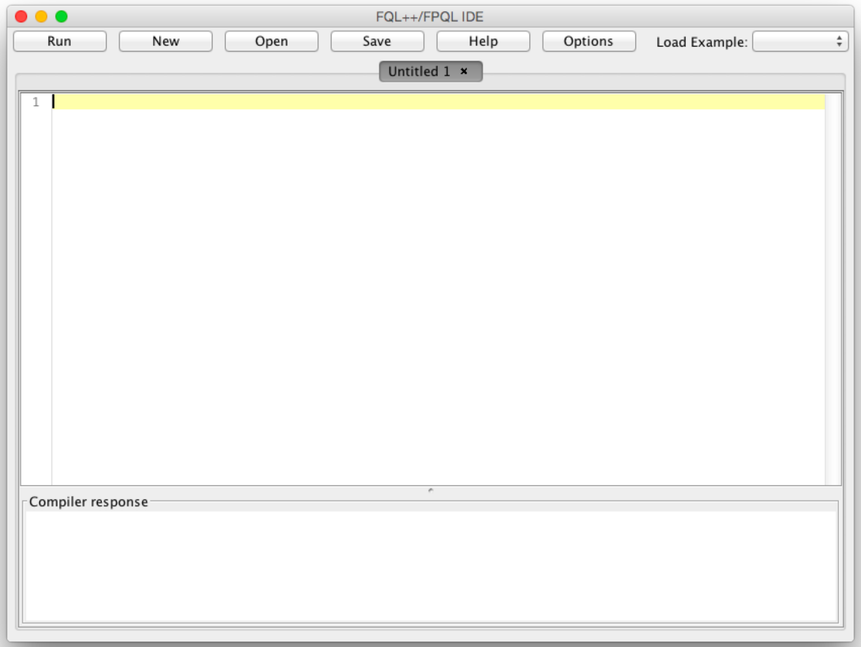
March 3, 2015

Contents

1 Introduction

FQL++, a successor to FQL, the *functorial query language*, implements *functorial data migration*. The FQL++ IDE is an open-source java program that provides a code editor for and visual representation of FQL++ programs. A screen shot of the initial screen of the FQL++ IDE is shown below. As FQL++ succeeds FQL, readers are recommended to first familiarize themselves with FQL and the FQL IDE.



The FQL++ IDE is a multi-tabbed text file editor that supports saving, opening, copy-paste, etc. Associated with each editor is a "compiler response" text area that displays an error message if execution fails. The built-in FQL++ examples can be loaded by selecting them from the "load example" combo box in the upper-right. In the rest of this tutorial we will refer to these examples. Compilation can be aborted with the "abort" button in the Tools menu. Abort is "best effort" and can leave FQL++ in an inconsistent state; it is provided to allow users to terminate FQL++ gracefully.

To run FQL++ with more than the default 64mb heap, you must use command line options:

```
java -Xms512m -Xmx2048m -jar fql_lib.jar
```

2 FQL++ Basics

Unlike FQL, which focuses on SQL compatibility and "attributes", the focus of FQL++ is on the "pure" functorial data model. That is, the core concepts in FQL++ are *categories*, *functors*, and *natural transformations*, and an FQL++ program is an ordered list of uniquely named *declarations* of these three kinds. Comments in FQL++ are Java style, either "//" or "/* */". Negative integers must be quoted with double quotes.

2.1 Categories

Select the "employees" examples. This action will create a new tab containing the following FQL++ code:

```
category S = {
 objects
  Employee, Department;
 arrows
  manager    : Employee -> Employee,
  worksIn    : Employee -> Department,
  secretary : Department -> Employee;
 equations
   Employee.manager.worksIn = Employee.worksIn,
   Department.secretary.worksIn = Department,
   Employee.manager.manager = Employee.manager;
}
```

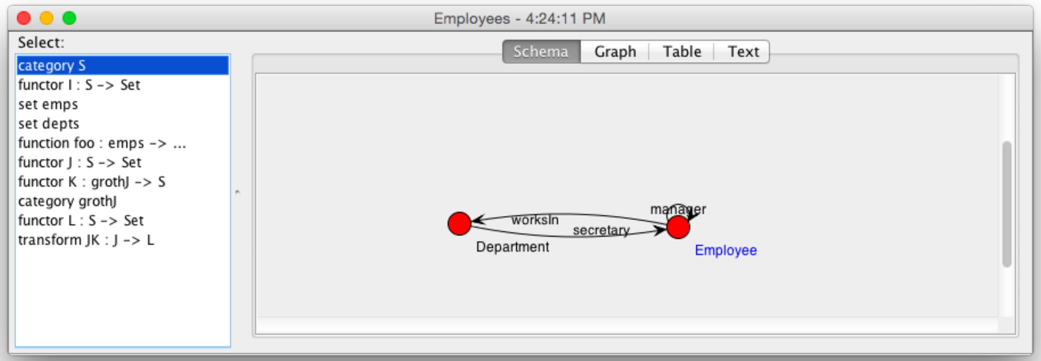
This declaration defines a category presentation S consisting of two *objects*, three *arrows*, and three *equations.* Unlike FQL, in FQL++ *there are no attributes.* In relational terminology, this means that

- Each node corresponds to an *entity type.* In this example, the two types of entities are employees and departments.
- Each arrow $f : X \to Y$ corresponds to a total function f from entities of type X to entities of type Y. In this example, manager maps employees to employees, worksIn maps employees to departments, and secretary maps departments to employees.
- The equations specify the data integrity constraints that must hold of all instances that conform to this schema. FQL++ uses equalities of paths as constraints. A path p is defined inductively as

  $$p ::= node \mid p.arrow$$

  Intuitively, the meaning of "." is composition. In this example, the constraints are: 1) every employee must work in the same department as his or her manager; 2) every departmental secretary must work for that department; and 3) there are employees and managers, but not managers of managers, managers of managers of managers, etc.

The FQL++ IDE can render categories into a graphical form. Press “run”, and select the category S from the viewer:



Note that the three sections, “objects”, “arrows”, and “equations” are ended with semi-colons, and must appear in that order, even when a section is empty. The “text” tab prints the category that the presentation denotes.

2.1.1 Infinite Categories

FQL++ requires all category presentations to denote finite categories. Such category presentations can be cyclic, provided they have “finitizing” equations, such as above.

2.2 Functors

Next, load the “delta” example. It defines two categories, C and D, and a functor F from C to D:

```
 category C = {
 objects
T1, T2, string, int;
 arrows
t1_ssn     : T1 -> string,
t1_first   : T1 -> string,
t1_last    : T1 -> string,
t2_first   : T2 -> string,
t2_last    : T2 -> string,
t2_salary : T2 -> int;
 equations;
}
```

$\rightarrow_F$

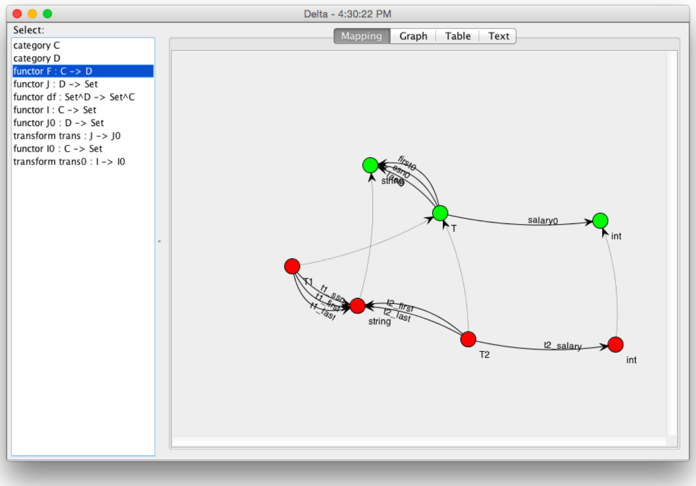
```
category D = {
 objects
T, string, int;
 arrows
ssn0     : T -> string,
first0   : T -> string,
last0    : T -> string,
salary0 : T -> int;
 equations;
}
```

```
functor F = {
 objects
T1 -> T,
T2 -> T,
string -> string,
int -> int;
 arrows
t1_ssn    -> T.ssn0,
t1_first  -> T.first0,
t2_first  -> T.first0,
t1_last   -> T.last0,
t2_last   -> T.last0,
t2_salary -> T.salary0;
} : C -> D
```

A functor $F : C \to D$ between finitely presented C and D consists of two parts:

- a mapping from the objects in C to the objects in D
- a mapping from the arrows in C to paths in D

A functor must respect the equations of C and D: if p_1 and p_2 are equal paths in C, then $F(p_1)$ and $F(p_2)$ must be equal paths in D. If this condition is not met, FQL++ will throw an exception. Our example mapping is rendered in the viewer as follows:



An identity mapping can be formed using the keyword “id” as follows:

```
functor F = id C
```

Mappings can be composed using the keyword “;” (parenthesis required) as follows:

```
functor F = (G ; H)
```

2.2.1 Instances

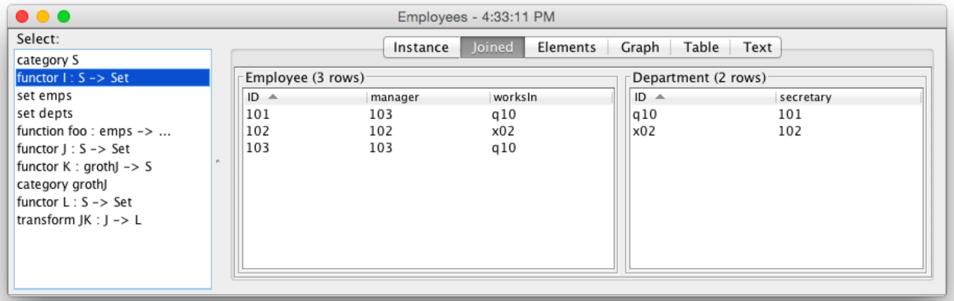
Continuing with the built-in "employees" example, we see that it also contains FQL++ code that defines an instance – a set-valued functor – of the category S defined previously:

```
functor I = {
 objects
  Employee -> { 101, 102, 103 },
  Department -> { q10, x02 };
 arrows
  manager -> { (101, 103), (102, 102), (103, 103) } : {101,102,103} -> {101,102,103},
  worksIn -> { (101, q10), (102, x02), (103, q10) } : {101,102,103} -> {q10,x02},
  secretary -> { (q10, 101), (x02, 102) } : {q10,x02} -> {101,102} ;
} : S -> Set
```

This declaration defines an instance I that conforms to category S. This means that

- To each node/entity type corresponds a set of IDs. In this example, the employee IDs are 101, 102, and 103, and the departmental IDs are q10 and x02.
- Each arrow $f : X \to Y$ corresponds to a function that maps IDs of entity type X to IDs of entity type Y.

To visualize this instance, press "run", select the instance from the viewer list, and click the "joined" tab:



Employee (3 rows)

ID	manager	worksIn
101	103	q10
102	102	x02
103	103	q10

Department (2 rows)

ID	secretary
q10	101
x02	102

To enter a string that contains spaces, simply quote it.

2.2.2 Category of Elements

Provided the "Elements" option is enabled, the "Elements" tab in an instance displays the instance as a category, "the category of its elements". In this view, nodes are entities and arrows are foreign-key correspondences.

2.2.3 Natural Transformations (database homomorphisms)

For each category presentation T and T-instances I,J, a *(natural) transformation* $f : I \Rightarrow J$ is a database homomorphism from I to J: for each node $n \in T$, f_n is a constraint respecting function from I_n IDs to J_n IDs. Transformations are illustrated in the "Delta" example.

```
functor J = {
 objects
  T -> { XF667, XF891, XF221 },
  string -> { "115-234", "112-988", "198-887", Bob, Sue, Alice, Smith, Jones },
  int -> { 250, 100, 300 };
 arrows
  ssn0    -> { (XF667, "115-234"),(XF891,"112-988"),(XF221,"198-887") },
  first0  -> { (XF667,Bob),(XF891,Sue),(XF221,Alice) },
  last0   -> { (XF667,Smith),(XF891,Smith),(XF221,Jones) },
  salary0 -> { (XF667,250),(XF891,300),(XF221,100) };
} : D -> Set

functor J0 = {
 objects
  T -> { XF66,XF89,XF22, xxx },
  string -> { "115-23", "112-98", "198-88", Bo, Su, Alic, Smit, Jone, xxx },
  int -> { 25, 10, 30, 0 };
 arrows
  ssn0    -> { (XF66, "115-23"),(XF89,"112-98"),(XF22,"198-88"), (xxx,"xxx") },
  first0  -> { (XF66,Bo),(XF89,Su),(XF22,Alic),(xxx, "xxx" )},
  last0   -> { (XF66,Smit),(XF89,Smit),(XF22,Jone), (xxx, "xxx") },
  salary0 -> { (XF66,25),(XF89,30),(XF22,10), (xxx, 0) };
} : D -> Set

transform trans = {
 objects
  T -> {(XF667,XF66),(XF891,XF89),(XF221,XF22)},
  string -> { ("115-234","115-23"), ("112-988","112-98"), ("198-887","198-88"),
              (Bob,Bo), (Sue,Su), (Alice,Alic), (Smith,Smit), (Jones,Jone) },
  int -> {(250,25),(100,10),(300,30)};
} : (J: D -> Set) -> (J0: D -> Set)
```

Note that the types of J and J_0 are required when defining *trans* above, and that $\rightarrow$ is used in FQL++ code rather than $\Rightarrow$.

2.3 Data Migration

Associated with a functor between finitely presented categories $F : C \to D$ are three data migration operators:

- Δ_F, taking D instances to C instances, roughly corresponding to projection
- Π_F, taking C instances to D instances, roughly corresponding to join
- Σ_F, taking C instances to D instances, roughly corresponding to union

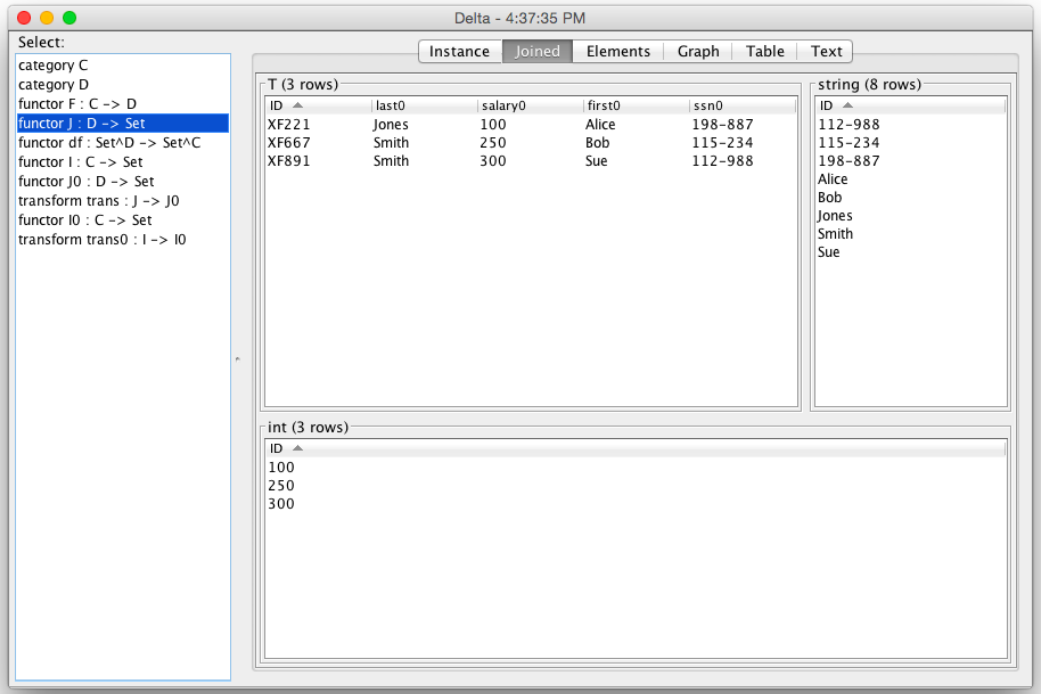
2.3.1 Delta

Continuing with the "delta" example, we see that the FQL++ program also defines a D-instance J, and computes $I := \Delta_F(J)$. In effect, we have projected the columns salary0 and last0 from J.

```
functor df = delta F

functor I = apply df on object J
```

Graphically, we have



Select:

category C
category D
functor F : C -> D
functor J : D -> Set
functor df : Set^D -> Set^C
functor I : C -> Set
functor J0 : D -> Set
transform trans : J -> J0
functor I0 : C -> Set
transform trans0 : I -> I0

T (3 rows)

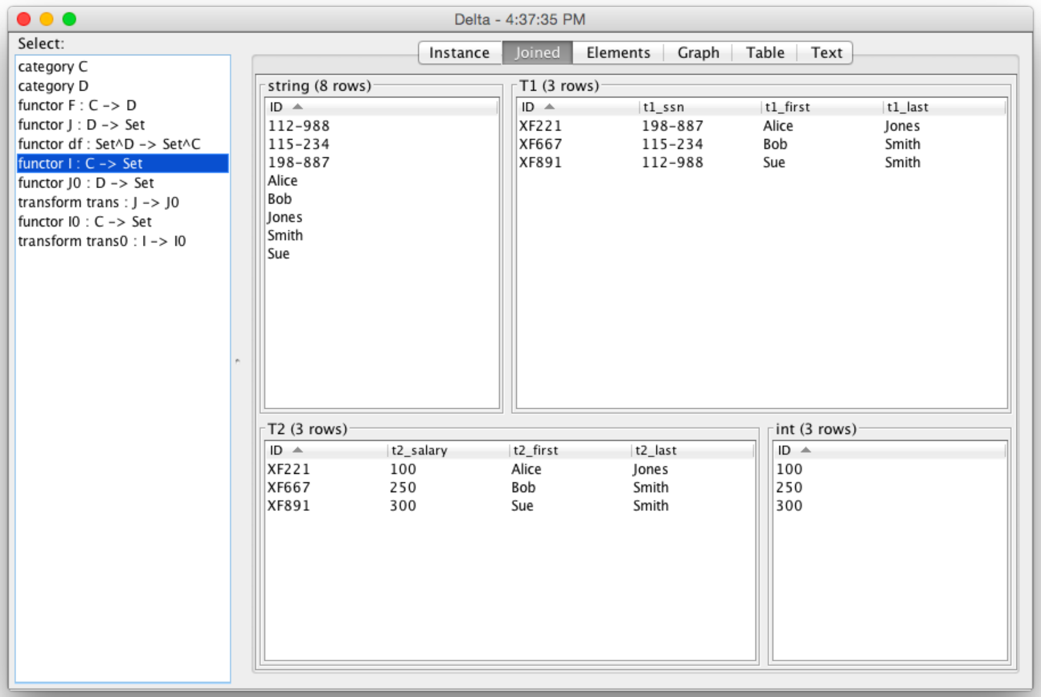
ID	last0	salary0	first0	ssn0
XF221	Jones	100	Alice	198-887
XF667	Smith	250	Bob	115-234
XF891	Smith	300	Sue	112-988

string (8 rows)

ID
112-988
115-234
198-887
Alice
Bob
Jones
Smith
Sue

int (3 rows)

ID
100
250
300



```
category C
category D
functor F : C -> D
functor J : D -> Set
functor df : Set^D -> Set^C
functor I : C -> Set
functor J0 : D -> Set
transform trans : J -> J0
functor I0 : C -> Set
transform trans0 : I -> I0
```

string (8 rows)

ID
112-988
115-234
198-887
Alice
Bob
Jones
Smith
Sue

T1 (3 rows)

ID	t1_ssn	t1_first	t1_last
XF221	198-887	Alice	Jones
XF667	115-234	Bob	Smith
XF891	112-988	Sue	Smith

T2 (3 rows)

ID	t2_salary	t2_first	t2_last
XF221	100	Alice	Jones
XF667	250	Bob	Smith
XF891	300	Sue	Smith

int (3 rows)

ID
100
250
300

2.3.2 Pi

Load the "pi" example:

```
  category C = {
  objects
   c1,
   c2,
   string;
  arrows
 att1 : c1 -> string,
 att2 : c1 -> string,
 att3 : c2 -> string;
  equations;
 }
```

$\rightarrow_F$

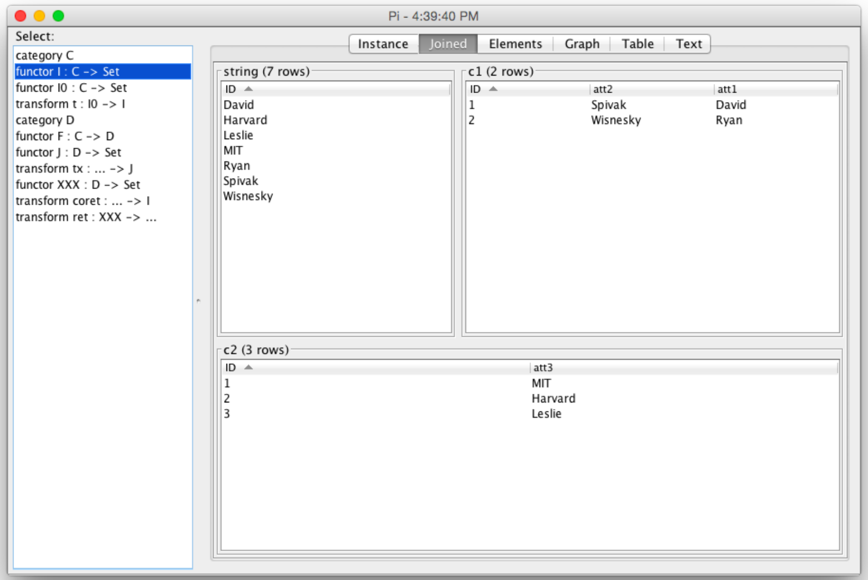
```
category D = {
 objects
  d,
  string;
 arrows
  a1 : d -> string,
  a2 : d -> string,
  a3 : d -> string;
 equations;
}
```

```
functor F = {
 objects
  c1 -> d, c2 -> d, string -> string;
 arrows
  att1 -> d.a1,  att2 -> d.a2, att3 -> d.a3;
} : C -> D
```

This example defines an instance $I : C$ and computes $J := \Pi_F(I)$:

```
functor J = apply pi F on object I
```

Graphically, this is rendered as:



string (7 rows)

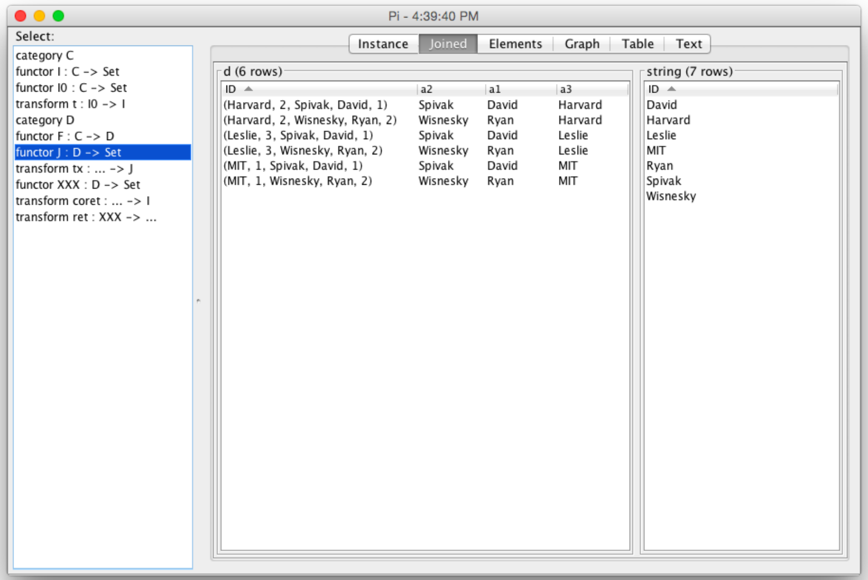
ID
David
Harvard
Leslie
MIT
Ryan
Spivak
Wisnesky

c1 (2 rows)

ID	att2	att1
1	Spivak	David
2	Wisnesky	Ryan

c2 (3 rows)

ID	att3
1	MIT
2	Harvard
3	Leslie



d (6 rows)

ID	a2	a1	a3
(Harvard, 2, Spivak, David, 1)	Spivak	David	Harvard
(Harvard, 2, Wisnesky, Ryan, 2)	Wisnesky	Ryan	Harvard
(Leslie, 3, Spivak, David, 1)	Spivak	David	Leslie
(Leslie, 3, Wisnesky, Ryan, 2)	Wisnesky	Ryan	Leslie
(MIT, 1, Spivak, David, 1)	Spivak	David	MIT
(MIT, 1, Wisnesky, Ryan, 2)	Wisnesky	Ryan	MIT

string (7 rows)

ID
David
Harvard
Leslie
MIT
Ryan
Spivak
Wisnesky

We see that we have computed a cartesian product. Note that unlike FQL, the IDs created by FQL++ have structure. The exact way to create new IDs is set by a menu option: fresh, corresponding to new IDs, summary, shown above, and lineage, which is the most verbose.

2.3.3 Sigma

Load the “sigma” example:

```
category C = {
objects
Amphibian,
LandAnimal,
WaterAnimal,
string;
arrows
attA : Amphibian -> string,
attL:LandAnimal-> string,
attW:WaterAnimal->string,
IsAL:Amphibian->LandAnimal,
IsAW:Amphibian->WaterAnimal;
equations;
}
```

$\to_F$

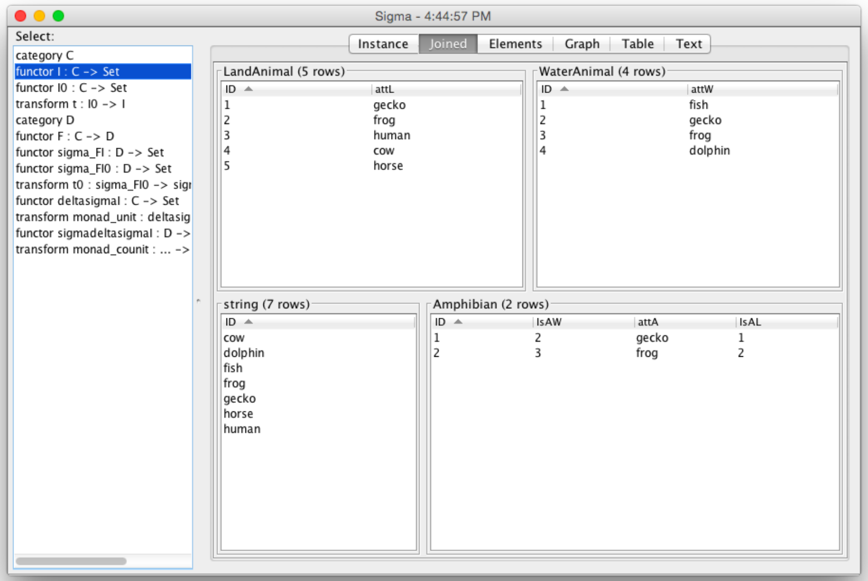
```
category D ={
objects
yAmphibian,
yLandAnimal,
yWaterAnimal,
yAnimal,
ystring;
arrows
yattA:yAmphibian -> ystring,
yattL:yLandAnimal-> ystring,
yattW:yWaterAnimal->ystring,
yIsAL:yAmphibian->yLandAnimal,
yIsAW:yAmphibian->yWaterAnimal,
yIsALL:yLandAnimal->yAnimal,
yIsAWW:yWaterAnimal->yAnimal;
equations
yAmphibian.yIsAL.yIsALL=
yAmphibian.yIsAW.yIsAWW;
}
```

```
functor F = {
objects
Amphibian->yAmphibian,
LandAnimal->yLandAnimal,
WaterAnimal->yWaterAnimal,
string->ystring;
arrows
attA -> yAmphibian.yattA,
attL -> yLandAnimal.yattL,
attW -> yWaterAnimal.yattW,
IsAL -> yAmphibian.yIsAL,
IsAW -> yAmphibian.yIsAW;
} : C -> D
```

This example defines an instance $I : C$ and computes $\Sigma_F(I)$:

```
functor sigma_FI = apply sigma F on object I
```

Graphically, this is rendered as:


Sigma - 4:44:57 PM
Select:
category C
functor I : C -> Set
functor I0 : C -> Set
transform t : I0 -> I
category D
functor F : C -> D
functor sigma_FI : D -> Set
functor sigma_FI0 : D -> Set
transform t0 : sigma_FI0 -> sigı
functor deltasigmaI : C -> Set
transform monad_unit : deltasig
functor sigmadeltasigmaI : D ->
transform monad_counit : ... ->
Instance
Joined
Elements
Graph
Table
Text

LandAnimal (5 rows)

ID	attL
1	gecko
2	frog
3	human
4	cow
5	horse

WaterAnimal (4 rows)

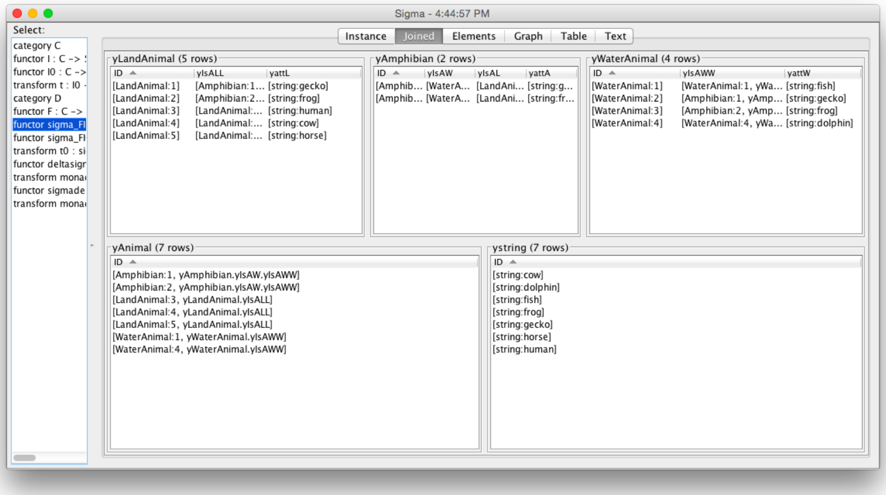
ID	attW
1	fish
2	gecko
3	frog
4	dolphin

string (7 rows)

ID
cow
dolphin
fish
frog
gecko
horse
human

Amphibian (2 rows)

ID	IsAW	attA	IsAL
1	2	gecko	1
2	3	frog	2



yLandAnimal (5 rows)

ID	yIsALL	yattL
[LandAnimal:1]	[Amphibian:1...	[string:gecko]
[LandAnimal:2]	[Amphibian:2...	[string:frog]
[LandAnimal:3]	[LandAnimal:...	[string:human]
[LandAnimal:4]	[LandAnimal:...	[string:cow]
[LandAnimal:5]	[LandAnimal:...	[string:horse]

yAmphibian (2 rows)

ID	yIsAW	yIsAL	yattA
[Amphib...	[WaterA...	[LandAni...	[string:g...
[Amphib...	[WaterA...	[LandAni...	[string:fr...

yWaterAnimal (4 rows)

ID	yIsAWW	yattW
[WaterAnimal:1]	[WaterAnimal:1, yWa...	[string:fish]
[WaterAnimal:2]	[Amphibian:1, yAmp...	[string:gecko]
[WaterAnimal:3]	[Amphibian:2, yAmp...	[string:frog]
[WaterAnimal:4]	[WaterAnimal:4, yWa...	[string:dolphin]

yAnimal (7 rows)

ID
[Amphibian:1, yAmphibian.yIsAW.yIsAWW]
[Amphibian:2, yAmphibian.yIsAW.yIsAWW]
[LandAnimal:3, yLandAnimal.yIsALL]
[LandAnimal:4, yLandAnimal.yIsALL]
[LandAnimal:5, yLandAnimal.yIsALL]
[WaterAnimal:1, yWaterAnimal.yIsAWW]
[WaterAnimal:4, yWaterAnimal.yIsAWW]

ystring (7 rows)

ID
[string:cow]
[string:dolphin]
[string:fish]
[string:frog]
[string:gecko]
[string:horse]
[string:human]

As with Pi, there are three ways for FQL++ to create fresh IDs in the target instance, and these are configured in the options menu.

2.3.4 Data Migration on Transformations

If $f : I \Rightarrow J$ is a transformation, then so is $\Delta_F(f) : \Delta_F(J) \Rightarrow \Delta_F(I)$, $\Sigma_F(f) : \Sigma_F(I) \Rightarrow \Sigma_F(J)$, $\Pi_F(f) : \Pi_F(f) : \Pi_F(I) \Rightarrow \Pi_F(J)$. See the All Syntax example for details.

2.3.5 Units and Co-units

For every F and I, there are canonical transforms $return : I \Rightarrow \Delta_F(\Sigma_F(I))$ and $coreturn : \Sigma_F(\Delta_F(I)) \Rightarrow I$, as well as canonical transforms $return : I \Rightarrow \Pi_F(\Delta_F(I))$ and $coreturn : \Delta_F(\Pi_F(I)) \Rightarrow I$. These transforms come from the mathematical theory of *monads*. They are illustrated in the Sigma and Pi examples.

2.4 Categorical Combinators

FQL++ contains programming capabilities in the guise of *categorical combinators.* Intuitively, these combinators are a standard functional programming language similar to e.g., point-free Haskell. Their semantics derives from the fact that the category of categories is bi-cartesian closed, and for each T, the category of T-instances is a topos. As these combinators are similar to the combinators in FQL, we do not repeat definitions here; instead, we just list all possible syntax. See the FQL IDE manual for descriptions of products, co-products, exponentials, and sub-object classifiers.

2.4.1 Programming with Categories and Functors

```
category c1 = void
category c2 = unit
category c3 = (c2 + c2)
category c4 = (c2 * c2)
category c5 = (unit ^ void)
category c6 = Set
category C  = {objects; arrows; equations;}

functor f1 = id unit
functor f2 = (f1 ; f1)
functor f4 = iso1 c4 c5
functor f6 = iso2 c4 c5
functor f7 = (fst c3 c3 * snd c3 c3)
functor f8  = (inl c3 c3 + inr c3 c3)
functor f9  = curry eval c3 c3
functor f10 = {objects; arrows;} : C -> C //mapping
functor f20 = {objects; arrows;} : C -> Set //instance
functor f13 = (f20 ^ f20)
functor f14 = delta f10
functor f15 = sigma f10
functor f16 = pi f10
functor f17 = apply f14 on object f20
functor f18 = prop C
functor t21 = void C Set //need Set here
functor t22 = unit C Set //need Set here
functor f3 = ff unit
functor f5 = tt unit

category c6x = dom f1
category c7x = cod f2
```

2.4.2 Programming with Functors and Transformations

```
transform t0 = {objects;} : (f20:C->Set) -> (f20:C->Set)
//must have dom and cod of functors above
transform t1 = id f20
transform t2 = (t1 ; t1)
transform t3x= (inl f20 f20 + inr f20 f20)
transform t4x= (fst f20 f20 * snd f20 f20)
transform t5 = curry eval f20 f20
transform t5x = tt f20
transform t5y = ff f20
transform t6 = apply f14 on arrow t1
transform t7 = true C
transform t8 = false C
transform t9 = and C
transform t7y= or C
transform t7x= not C
transform t8z= implies C
transform t8x= return sigma delta f10
transform t9x= coreturn sigma delta f10
transform t10= return delta pi f10
transform t11= coreturn delta pi f10
transform t12= iso1 f20 f20
transform t13= iso2 f20 f20

functor f19x = dom t1
functor t20x = cod t1
```

3 FQL++ features beyond FQL

The fragment of FQL++ described above is a subset of FQL: the subset containing no attributes. In exchange for removing attributes, FQL++ contains a number of features that go beyond FQL.

3.1 Programming with Sets

FQL++ contains a full programming language for finite sets based on the internal language of a BCCC. See the 'Sets' example. These sets and functions can be used anywhere that FQL++ expects a set or function, for example, in an instance declaration.

```
/*
 * In FQL++, a *value* is either:
 *
 *  - a string, that must be quoted if it contains spaces or special symbols, like "-".
 *        Numbers are treated as strings in FQL++.
 *  - a unit, written ().
 *  - a boolean, written true or false.
 *  - a pair of two values v1 and v2, written (v1, v2).
 *  - a left or right injection of a value v, written inl v or inr v.
 *  - a (possibly empty) set of values, written {v1, ..., vn}.
 *  - a function, written as a set of pairs with explicit domain and co-domain:
 *        {(x1,y1), ..., (xN,yN)} : {x1, ..., xN} -> {y1, ..., yN}.
 */

//A *set literal* is a set of values, for example:
set s1 = {foo, bar, "b a z", 7, true, false}
set s2 = {(), inl (), inr (), inl inr 7, (hello,world), {a,b,{c,d},{}}}
set s3 = { {(1,1),(2,4),(3,9)} : {1,2,3} -> {1,4,9}, {(1,1),(2,4),(3,9)}
  : {1,2,3} -> {1,4,9,10,11,12} }

//A *set* is either a set literal, as above, or a set constructed from
//set literals and +,*,^,void,unit,prop, for example:
set s4 = void // = {}
set s5 = unit // = { () }
set s6 = prop // = { true, false }
set s7 = (unit + unit) // = { inl (), inr () }
set s8 = (prop * prop) // = { (true,true),(true,false),(false,true),(false,false) }
set s9 = (prop ^ unit) // = { ((), true) : {()} -> {true,false}, ((), false)
  : {()} -> {true,false} }

//a *function literal* is a function value, for example
function f1 = { (1,2),(2,3) } : {1,2} -> {2,3}
```

```
//functions can refer to named sets, so we could write instead
set sf = {1,2}
function f2 = { (1,2),(2,3) } : sf -> {2,3}

//a *function* is either a function literal, as above, or a function constructed from
//function literals and tt,ff,fst,snd,(+),(*),(;),eval,curry,char,kernel,iso1,iso2,id
//for example
function f3 = fst {a,b} {c,d} // (a,c) -> a  (a,d) -> a  (b,c) -> b  (b,d) -> d
function f4 = inl {a,b} {c,d} // a -> inl a, b -> inl b
function f5 = id {a,b} // a -> a, b -> b
function f6 = (f5 ; f5) // = f5
function pair_eta = (fst {a,b} {c,d} * snd {a,b} {c,d}) // = id
function sum_eta = (inl {a,b} {c,d} + inr {a,b} {c,d}) // = id
function exp_eta = curry eval {a} {b} // = id
function f7 = char {(1,a),(2,b)} : {1,2} -> {a,b,c} // = a -> true, b -> true
function f8 = kernel f7 // = {(a, a),  (b, b)} : {a, b} -> {a, b, c}
function f9 = iso1 {a,b} {1,2} // = a -> 1, b -> 2
function f10 = tt {a}
function f11 = ff {a}
function f12 = true
function f13 = false
function f14 = and
function f15 = or
function f16 = not
function f17 = implies

//Domains, co-domains, and range require that you must name the function, i.e., as f7
set x = dom f7 // = {a,b}
set y = cod f7 // = {true,false}
set z = range f8 // = {a,b}

//the number n denotes the set {0, ..., n-1}
set two = 2
set three = 3
set five = 5
set two_plus_three = (2+3)
function two_plus_three_equals_five = iso1 two_plus_three five
```

3.2 The categories *Cat*, *Set*, and *C*-Inst

In addition to finitely presented categories, FQL++ contains keywords for the category of sets, *Set*, the category of categories, *Cat*, and for each finitely presented C, the category Set^C. It is possible to create functors from finitely presented categories to these categories, and, moreover, to create transforms between such functors. See the “categories” example.

```
//////////////////
//cat-valued functors
//let C and D be finitely presented categories, and F a mapping
category X = {
 objects
c, d;
 arrows
f : c -> d;
 equations;
}

functor Y = {
 objects
     c -> C,
     d -> D;
 arrows
f -> F;
} : X -> Cat

/////////////////////////////////////////////
//Transforms between functors to Set

functor X1 = {
 objects
  c -> {1,2},
  d -> {3,4};
 arrows
  f -> {(1,3),(2,4)};
} : X -> Set

functor X2 = {
 objects
  c -> {a,b,c},
  d -> {d,e,f};
 arrows
  f -> {(a,d),(b,e),(c,f)};
} : X -> Set

transform X1X2 = {
 objects
  c -> {(1,a),(2,b)},
  d -> {(3,d),(4,e)};
} : (X1 : X -> Set) -> (X2 : X -> Set)
```

```
/////////////////////////////////////////////
//Transforms between functors to Cat

functor Z = {
 objects
     c -> C,
     d -> C;
 arrows
f -> id C;
} : X -> Cat

transform ZY = {
 objects
  c -> id C,
  d -> F;
} : (Z : X -> Cat) -> (Y : X -> Cat)


///////////////////////////////////////////////////////////////////////
//Transforms between functors between finitely presented categories

transform FF = {
 objects
  T1 -> T,
  T2 -> T,
  string -> string,
  int -> int;
} : (F : C -> D) -> (F : C -> D)
```

A particularly useful application of *Cat*-valued functors is the ability to check commutative diagrams in Cat; see the 'Integration' example.

3.3 Functors $Set \to Set$, and Monads

FQL++ has the ability to define functors $Set \to Set$. Given a monad (on Set, or on a finitely presented category), FQL++ can create the associated Kleisli and co-Kleisli categories. See the 'Monads' example.

```
//functors Set -> Set
functor Maybe = {
 objects
  X -> (unit + X);
 arrows
  f : A -> B -> (inl unit B + (f ; inr unit B));
} : Set -> Set
```

```
set Maybe3 = apply Maybe on object 3
set Maybe4 = apply Maybe on object 4

function MaybeF = apply Maybe on arrow {(0,1),(1,2),(2,0),(3,0)} : 4 -> 3

functor Double = {
objects X->(X*X);
arrows f:A->B -> ((fst A A; f)*(snd A A; f));
} : Set -> Set

set dbl=apply Double on object {1,2,3}

function f = apply Double on arrow id {1,2,3}

transform ret = {
 objects
  X -> inr unit X;
} : (id Set : Set -> Set) -> (Maybe : Set -> Set)

transform join = {
 objects
  X -> (inl unit X + id (unit + X));
} : ((Maybe ; Maybe) : Set -> Set) -> (Maybe : Set -> Set)

function ret1 = apply ret {1,2,3}
set set1 = apply Maybe on object {1,2,3}
set set2 = apply Maybe on object set1
function joined = apply join set2

category K = kleisli Maybe ret join

////////

category C = {
objects a,b,c;
arrows f:a->b,g:b->c;
equations;
}

functor M = {
objects a->a, b->c,c->c;
arrows f->a.f.g,g->c;
} : C -> C
```

```
transform eta = {
objects a->a, b->b.g, c->c;
} : (id C:C->C) -> (M: C->C)

transform mu = {
objects a->a, b->c, c->c;
} : ((M;M):C->C) -> (M: C->C)

category M_kleisli = kleisli M eta mu

///////////

category D = {
objects a,b,c;
arrows f:a->b,g:b->c;
equations;
}

functor T = {
objects a->a, b->a,c->c;
arrows f->a,g->a.f.g;
} : D -> D

transform epsilon = {
objects a->a, b->a.f, c->c;
} : (T: D->D) -> (id D:D->D)

transform Delta = {
objects a->a, b->a, c->c;
} : (T:D->D) -> ((T;T): D->D)

category CK = cokleisli T epsilon Delta
```

3.4 Extended Currying

FQL++ can convert a functor $A \times B \to Set$ to a functor $A \to Set^B$ using the keyword CURRY. See the integration example.

3.5 Pivot and Un-pivot

FQL++ can convert a set-valued functor into a category and vice-versa. See the Employees example.